Duration: 3 Hours Marks: 80

N.B: a) Question number 1 is compulsory

b) Solve any three from the remaining.

c) All the question carry equal marks

1. a) Find the extremal of $\int_0^{\pi} \frac{1+y^2}{y'^2} dx$ subject to $y(0) = 0, y(\pi) = 0$. [5]

b) Using Cauchy's Schwartz Inequality, show that $(a cos\theta + b sin\theta)^2 \le a^2 + b^2$, Where 'a' and 'b' are real. [5]

c) Show that Eigen values of Hermitian matrix are real. [5]

d) Evaluate $\int (z^2 - 2\bar{z} + 1)\, dz$ over a closed circle $x^2 + y^2 = 2$. [5]

2. a) Find the extremal $\int_{x_1}^{x_2} (y^2 - y'^2 - 2y cosh x) dx$ [6]

b) Find the Eigen values and Eigen Vectors of the matrix $A^2 + 3I$, where [6]

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

c) Obtain all possible expansion of $f(z) = \frac{1}{z^2(z-1)(z+2)}$ about $z = 0$ indicating region of convergence. [8]

3. a) Verify Cayley - Hamilton Theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and find A^{-1}. [6]

b) b) Using Residue theorem evaluate $\int_C \frac{e^z}{z^2 + \pi^2} dz$ where C is |z|=4. [6]

c) Show that a closed curve 'C' of a given fixed length (perimeter) which encloses maximum area is a circle. [8]

4. a) Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process, where $u_1 = (1,0,0), u_2 = (3,7,-2), u_3 = (0,4,1)$. [6]

b) Find A^{50} for the matrix $A = \begin{bmatrix} 4 & 3 \\ 7 & 8 \end{bmatrix}$ [6]

c) Reduce the Quadratic Form $xy + yz + zx$ to normal form by congruent transformation. [8]

5. a) Using Rayleigh-Ritz Method, find an approximate solution to the extremal problem $\int_0^1 (y^2 + 2yx - y'^2)dx, \quad y(0) = 0,\ y(1) = 0.$ [6]

b) Determine whether the set $V = \{(x, y, z): x = 1, y = 0 \; or \; z = 0\}$ is a subspace of R^3 [6]

c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diogonable.Also find the transforming matrix and diagonal matrix. [8]

6. a) Using Cauchy's Residue Theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{2+ COS\theta}$ [6]

b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy)dz$ along the straight line joining $A(1, -1)$ and $B(2,1)$ [6]

c) Find the singular value decomposition of the matrix A= $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ [8]

****